Quantitative Stream Reasoning with LARS

Rafael Kiesel, Thomas Eiter

Vienna University of Technology
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(Qualitative) Stream Reasoning with LARS

▶ Does a tram arrive at station $s$ within the next 20 minutes?
(Qualitative) Stream Reasoning with LARS

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→ $\square +^{20} \Diamond Tram(X, s)$
(Qualitative) Stream Reasoning with LARS

- Does a tram arrive at station $s$ within the next 20 minutes?
- $\square^{+20}\diamondsuit Tram(X, s)$
- Can I go from station $s$ to another station $s'$ using a tram that arrives within 15 minutes?
(Qualitative) Stream Reasoning with LARS

- Does a tram arrive at station \( s \) within the next 20 minutes?
  \[ \square^{+20} \Diamond Tram(X, s) \]

- Can I go from station \( s \) to another station \( s' \) using a tram that arrives within 15 minutes?
  \[ After(s, s') \land \square^{+15} \Diamond Tram(X, s) \land \neg Full(X) \]
Quantitative?

- How many trams will arrive at station $s$ within the next 20 minutes?
Quantitative?

- How many trams will arrive at station $s$ within the next 20 minutes?
  - Expect answer in $\mathbb{N}$
Quantitative?

- How many trams will arrive at station $s$ within the next 20 minutes?
  - Expect answer in $\mathbb{N}$

- How likely is it that I can go from station $s$ to another station $s'$ using a tram that arrives within 15 minutes?
How many trams will arrive at station $s$ within the next 20 minutes?

→ Expect answer in $\mathbb{N}$

How likely is it that I can go from station $s$ to another station $s'$ using a tram that arrives within 15 minutes?

→ Expect answer in $[0, 1]$
Quantitative extensions of LARS

- Ad Hoc
- Framework
Our Work

- General framework
- Semirings as algebraic structure underlying calculations
- Introduce weighted LARS formulas (over semirings)
- Semantics assigns a numerical value (in the semiring)
- Applicability of our framework
Preliminaries

- Interpretations \((S, t)\), with \(S = (v, T)\) a stream consisting of an evaluation function \(v\) and a set \(T\) of time points that are considered, that contains the current time \(t\).

- Assign LARS formulas

\[
\alpha ::= p \mid \neg \alpha \mid \alpha \land \alpha \mid \alpha \lor \alpha \mid \diamond \alpha \mid \Box \alpha \mid \Theta_t \alpha \mid \Box^w \alpha
\]

a boolean value.

- Examples:

  - \(\diamond Tram(x, s)\)
  - \(\neg \Theta_T Tram(x, s) \lor \neg \Theta_{T+1} Tram(x, s)\)
Semiring

A semiring is an algebraic structure \((R, \oplus, \otimes, e_\oplus, e_\otimes)\), s.t.

- \((R, \oplus, e_\oplus)\) is a commutative monoid with neutral element \(e_\oplus\)
- \((R, \otimes, e_\otimes)\) is a monoid with neutral element \(e_\otimes\)
- multiplication \((e_\otimes)\) distributes over addition \((e_\oplus)\)
- multiplication by \(e_\oplus\) annihilates \(R\)
  \((\forall r \in R : e_\oplus \otimes r = e_\oplus = r \otimes e_\oplus)\)

Examples are

- \((\mathbb{N}, +, \cdot, 0, 1)\), the semiring over the natural numbers
- \(((0, 1], \max, \cdot, 0, 1)\), a probability semiring
- \(((\perp, \top}, \lor, \land, \perp, \top)\), a boolean algebra
Weighted LARS Syntax

We define weighted LARS formulas over a semiring $\mathcal{R} = (R, \oplus, \otimes, e_\oplus, e_\otimes)$ similarly to how weighted MSO formulas are defined in [Droste and Gastin2007]

$$\alpha ::= k \mid p \mid \neg \alpha \mid \alpha \land \alpha \mid \alpha \lor \alpha \mid \Diamond \alpha \mid \Box \alpha \mid \circ_t \alpha \mid \mathbb{F}^w \alpha,$$

where $k \in R$. 
Weighted LARS Semantics I

- **Goal:** Assign a formula a numerical value
- **Use** $e_\otimes$ and $e_\oplus$ as truth and falsehood respectively
- **Interpret** disjunction as sum and conjunction as product
- **Formally**, for an interpretation $(S, t)$, where $S = (v, T)$:

\[
\begin{align*}
\llbracket k \rrbracket_R(S, t) &= k, \text{ for } k \in R \\
\llbracket p \rrbracket_R(S, t) &= \begin{cases} 
    e_\otimes, & \text{if } p \in v(t) \\
    e_\oplus, & \text{otherwise.}
\end{cases} \\
\llbracket \alpha \land \beta \rrbracket_R(S, t) &= \llbracket \alpha \rrbracket_R(S, t) \otimes \llbracket \beta \rrbracket_R(S, t) \\
\llbracket \alpha \lor \beta \rrbracket_R(S, t) &= \llbracket \alpha \rrbracket_R(S, t) \oplus \llbracket \beta \rrbracket_R(S, t)
\end{align*}
\]
Weighted LARS Semantics II

- Negation is close to inversion of the truth value
- Interpret existential quantification as sum and universal quantification as product

\[
[-\alpha]_R(S, t) = \begin{cases} 
  e_\otimes, & \text{iff } [\alpha]_R(S, t) = e_\oplus \\
  e_\oplus, & \text{otherwise.}
\end{cases}
\]

\[
[\Diamond \alpha]_R(S, t) = \bigoplus_{t' \in T} [\alpha]_R(S, t')
\]

\[
[\Box \alpha]_R(S, t) = \bigotimes_{t' \in T} [\alpha]_R(S, t')
\]

\[
[\@_{t'} \alpha]_R(S, t) = [\alpha]_R(S, t')
\]

\[
[\Box^w \alpha]_R(S, t) = [\alpha]_R(\Box^w(S, t), t)
\]

Rafael Kiesel, Thomas Eiter
Example

How many trams will arrive at station $s$ within the next 20 minutes?
Example

- How many trams will arrive at station $s$ within the next 20 minutes?

$\Rightarrow \Box^{+20}\Diamond Tram(X, s)\over (\mathbb{N}, +, \cdot, 0, 1)$
Example

- How many trams will arrive at station $s$ within the next 20 minutes?
  \[\blacklozenge^20\Diamond Tram(X, s)\] over $(\mathbb{N}, +, \cdot, 0, 1)$

- How likely is it that I can go from station $s$ to another station $s'$ using a tram that arrives within 15 minutes?
Example

- How many trams will arrive at station $s$ within the next 20 minutes?
  $$\Box^{+20} \diamond Tram(X, s) \text{ over } (\mathbb{N}, +, \cdot, 0, 1)$$

- How likely is it that I can go from station $s$ to another station $s'$ using a tram that arrives within 15 minutes?
  $$After(s, s') \land \Box^{+15} \diamond Tram(X, s) \land \neg Full(X) \lor Tram(X, s) \land Full(X) \land 0.3 \text{ over } ([0, 1], \max, \cdot, 0, 1)$$
A LARS measure $\mu$ is defined by a triple $\langle \Pi, \alpha, \mathcal{R} \rangle$, where

- $\Pi$ is a LARS program
- $\alpha$ is a weighted LARS formula over $\mathcal{R}$
- $\mathcal{R}$ is a semiring

We set

$$
\mu(S, t) = \begin{cases} 
[\alpha]_{\mathcal{R}}(S, t) & \text{iff } S \text{ is an answer stream of } \Pi \text{ at } t, \\
e_{\oplus} & \text{otherwise.}
\end{cases}
$$
Problem definitions

- Optimisation:
  \[ \text{argmax}_{(S,t)} \mu(S, t) \]

- Probabilistic reasoning:
  \[ P_\mu(S, t) = \frac{\mu(S, t)}{\sum_{(S', t')} \mu(S', t')} \]
  \[ P_\mu(\phi, t) = \sum_{S, (S,t) \models \phi} P(S, t) \]
  \[ E_\mu[\beta] = \sum_{(S,t)} [\beta]_R(S, t) P(S, t) \]
Applications

- Problog [De Raedt et al. 2007]: Probabilistic reasoning. Can be expressed using the framework.
- LP$^{	ext{MLN}}$ [Lee and Yang 2017]: Probabilistic reasoning. Can be expressed with

\[ \mu(S, t) = \left\{ \begin{array}{ll} \lceil [\alpha] R(S, t) \rceil & \text{iff } S \text{ is an answer stream of } \Pi_{S,t} \text{ at } t, \\ e & \text{otherwise.} \end{array} \right. \]

Reliability of constraint satisfaction

- Using the semiring over the natural numbers, we can evaluate how many proofs there are for a formula.
- We consider answer streams of a program $\Pi$ more reliable if there are more proofs for a constraint $\alpha$
- Assume that the probability of an answer stream is proportional to the number of proofs for the constraint
  $\Rightarrow$ probability distribution given as $\mathbb{P}_\mu$ induced by $\langle \Pi, \alpha, \mathbb{N} \rangle$
Relation to other formalisms

- ASP expressible in second order logic
- for the propositional case even in monadic second order logic (MSO)
- Fragment of weighted MSO by [Droste and Gastin2007] equivalent to weighted automata
- Similarly a fragment of the problems definable using LARS measures is equivalent to weighted automata
Future/Ongoing work

- Weighted LARS formulas for aggregates, weighted constraints and more
- Implementation
- Complexity considerations
- General properties of extensions formalised using weighted formulas?
Questions?
References I


References II
