Quantitative Stream Reasoning with LARS

Rafael Kiesel, Thomas Eiter

Vienna University of Technology

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logics LOGICAL METHODS IN COMPUTER SCIENCE

(Qualitative) Stream Reasoning with LARS

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- Can I go from station s to another station s' using a tram that arrives within 15 minutes?

$\rightarrow$ $After(s, s') \land \boxplus^{+15}\Diamond Tram(X, s) \land \neg Full(X)$

Quantitative?

- How many trams will arrive at station *s* within the next 20 minutes?

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Quantitative?

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$\rightarrow$ Expect answer in $[0, 1]$

- ...

Quantitative?

Quantitative extensions of LARS

- Ad Hoc
- Framework

Our Work

- General framework
- Semirings as algebraic structure underlying calculations
- Introduce weighted LARS formulas (over semirings)
- Semantics assigns a numerical value (in the semiring)
- Applicability of our framework

Preliminaries

- Interpretations (S, t), with $S = (v, T)$ a stream consisting of an evaluation function v and a set T of time points that are considered, that contains the current time t.
- Assign LARS formulas

$$\alpha ::= p \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \Diamond\alpha \mid \Box\alpha \mid @_t\alpha \mid \boxplus^w\alpha$$

  a boolean value.
- Examples:
  - $\Diamond \mathit{Tram}(x, s)$
  - $\neg @_T \mathit{Tram}(x, s) \vee \neg @_{T+1} \mathit{Tram}(x, s)$

Semiring

A semiring is an algebraic structure $(R, \oplus, \otimes, e_\oplus, e_\otimes)$, s.t.

- $(R, \oplus, e_\oplus)$ is a commutative monoid with neutral element $e_\oplus$
- $(R, \otimes, e_\otimes)$ is a monoid with neutral element $e_\otimes$
- multiplication ($e_\otimes$) distributes over addition ($e_\oplus$)
- multiplication by $e_\oplus$ annihilates R
  $(\forall r \in R : e_\oplus \otimes r = e_\oplus = r \otimes e_\oplus)$

Examples are

- $(\mathbb{N}, +, \cdot, 0, 1)$, the semiring over the natural numbers
- $([0, 1], \max, \cdot, 0, 1)$, a probability semiring
- $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$, a boolean algebra

Weighted LARS Syntax

We define weighted LARS formulas over a semiring $\mathcal{R} = (R, \oplus, \otimes, e_\oplus, e_\otimes)$ similarly to how weighted MSO formulas are defined in [Droste and Gastin2007]

$$\alpha ::= k \mid p \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \Diamond\alpha \mid \Box\alpha \mid @_t\alpha \mid \boxplus^w\alpha,$$

where $k \in R$.

Weighted LARS Semantics I

- Goal: Assign a formula a numerical value
- Use $e_{\otimes}$ and $e_{\oplus}$ as truth and falsehood respectively
- Interpret disjunction as sum and conjunction as product
- Formally, for an interpretation (S, t), where $S = (v, T)$:

$$[\![k]\!]_{\mathcal{R}}(S, t) = k, \text{ for } k \in R$$

$$[\![p]\!]_{\mathcal{R}}(S, t) = \begin{cases} e_{\otimes}, & \text{if } p \in v(t) \\ e_{\oplus}, & \text{otherwise.} \end{cases}$$

$$[\![\alpha \wedge \beta]\!]_{\mathcal{R}}(S, t) = [\![\alpha]\!]_{\mathcal{R}}(S, t) \otimes [\![\beta]\!]_{\mathcal{R}}(S, t)$$

$$[\![\alpha \vee \beta]\!]_{\mathcal{R}}(S, t) = [\![\alpha]\!]_{\mathcal{R}}(S, t) \oplus [\![\beta]\!]_{\mathcal{R}}(S, t)$$

Weighted LARS Semantics II

- Negation is close to inversion of the truth value
- Interpret existential quantification as sum and universal quantification as product

$$\llbracket \neg\alpha \rrbracket_{\mathcal{R}}(S,t) = \begin{cases} e_{\otimes}, & \text{iff } \llbracket \alpha \rrbracket_{\mathcal{R}}(S,t) = e_{\oplus} \\ e_{\oplus}, & \text{otherwise.} \end{cases}$$

$$\llbracket \Diamond\alpha \rrbracket_{\mathcal{R}}(S,t) = \bigoplus_{t' \in T} \llbracket \alpha \rrbracket_{\mathcal{R}}(S,t')$$

$$\llbracket \Box\alpha \rrbracket_{\mathcal{R}}(S,t) = \bigotimes_{t' \in T} \llbracket \alpha \rrbracket_{\mathcal{R}}(S,t')$$

$$\llbracket @_{t'}\alpha \rrbracket_{\mathcal{R}}(S,t) = \llbracket \alpha \rrbracket_{\mathcal{R}}(S,t')$$

$$\llbracket \boxplus^{w}\alpha \rrbracket_{\mathcal{R}}(S,t) = \llbracket \alpha \rrbracket_{\mathcal{R}}(\boxplus^{w}(S,t),t)$$

Example

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- How likely is it that I can go from station s to another station s' using a tram that arrives within 15 minutes?

$\rightarrow$ $\mathit{After}(s, s') \wedge \boxplus^{+15}\Diamond \mathit{Tram}(X, s) \wedge \neg \mathit{Full}(X)$
$\vee \mathit{Tram}(X, s) \wedge \mathit{Full}(X) \wedge 0.3$
over $([0, 1], \max, \cdot, 0, 1)$

LARS measure

- A LARS measure μ is defined by a triple $\langle \Pi, \alpha, \mathcal{R} \rangle$, where
  - Π is a LARS program
  - α is a weighted LARS formula over $\mathcal{R}$
  - $\mathcal{R}$ is a semiring
- We set

$$\mu(S, t) = \begin{cases} [\![\alpha]\!]_{\mathcal{R}}(S, t) & \text{iff } S \text{ is an answer stream of } \Pi \text{ at } t, \\ e_{\oplus} & \text{otherwise.} \end{cases}$$

Problem definitions

- Optimisation:

$$\text{argmax}_{(S,t)}\, \mu(S,t)$$

- Probabilistic reasoning:

$$\mathbb{P}_\mu(S,t) = \frac{\mu(S,t)}{\sum_{(S',t')} \mu(S',t')}$$

$$\mathbb{P}_\mu(\phi,t) = \sum_{S,(S,t)\models\phi} \mathbb{P}(S,t)$$

$$\mathbb{E}_\mu[\beta] = \sum_{(S,t)} [\![\beta]\!]_{\mathcal{R}}(S,t)\mathbb{P}(S,t)$$

Applications

- P-log [Baral *et al.*2009]: Probabilistic reasoning. Can be expressed using the framework.
- Problog [De Raedt *et al.*2007]: Probabilistic reasoning. Can be expressed using the framework.
- LP^{MLN} [Lee and Yang2017]: Probabilistic reasoning. Can be expressed with

$$\mu(S,t) = \begin{cases} [\![\alpha]\!]_{\mathcal{R}}(S,t) & \text{iff } S \text{ is an answer stream of } \Pi_{S,t} \text{ at } t, \\ e_{\oplus} & \text{otherwise.} \end{cases}$$

- PrASP [Nickles and Mileo2015]: Probabilistic reasoning. No obvious relation to our framework.

Reliability of constraint satisfaction

- Using the semiring over the natural numbers, we can evaluate how many proofs there are for a formula.
- We consider answer streams of a program Π more reliable if there are more proofs for a constraint α
- Assume that the probability of an answer stream is proportional to the number of proofs for the constraint

$\rightarrow$ probability distribution given as $\mathbb{P}_\mu$ induced by $\langle \Pi, \alpha, \mathbb{N} \rangle$

Relation to other formalisms

- ASP expressible in second order logic
- for the propositional case even in monadic second order logic (MSO)
- Fragment of weighted MSO by [Droste and Gastin2007] equivalent to weighted automata
- Similarly a fragment of the problems definable using LARS measures is equivalent to weighted automata

Future/Ongoing work

- Weighted LARS formulas for aggregates, weighted constraints and more
- Implementation
- Complexity considerations
- General properties of extensions formalised using weighted formulas?

Questions?

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