Probabilistic Signal Temporal Logic for Predictive Stream Reasoning

Mattias Tiger  Fredrik Heintz

Artificial Intelligence and Integrated Computer Systems
Department of Computer and Information Science
Linköping University, Sweden
Unmanned Aerial Vehicle (UAV)
Stream Reasoning

Stream:

altitude

Altitude [meters]

3

Mattias Tiger
Linköping University
Stream Reasoning

Global constraint:

\( \Box (\text{altitude} > 3) \)

True or False

Monitoring:

altitude \( > 3 \)
Stream Reasoning

Metric Temporal Logic (MTL)

□ (altitude > 3)

Incremental Reasoning using Progression
Reason over predictions of the future

React when/after it happens:

□ (altitude > 3)
Reason over predictions of the future

React when/after it happens:

□ (altitude > 3)

React before it happens:

□ (altitude_{2|0} > 3)
Reason over predictions of the future

React when/after it happens:

□ (altitude > 3)

React before it happens:

□ (altitude_{2|0} > 3)

Collision?
Predicting the future?

Predicting future states

- What is the source of the stream and its states?
- How are states related over time?
Streams and Signals

**State Stream**

\[ \text{stream} \models \phi \quad \Rightarrow \quad \text{signal} \models \phi \]

**Continuous Signal**

**altitude**

**Discretiation**

**Physical Property**
**Definition (STL model)**

STL is defined over model $\mathcal{M} = \langle S, F_B \rangle$. $f_p \in F_B : \mathbb{R}^{|S|} \to \{ T, \bot \}$.

**Definition (STL syntax)**

$$\phi ::= T \mid p \mid \neg \phi \mid \phi \lor \psi \mid \phi \mathcal{U}_I \psi$$

**Definition (STL semantics)**

- $\mathcal{M}, n \models T$
- $\mathcal{M}, n \models p$ iff $f_p(S_n)$
- $\mathcal{M}, n \models \neg \phi$ iff $\mathcal{M}, n \not\models \phi$
- $\mathcal{M}, n \models \phi \lor \psi$ iff $\mathcal{M}, n \models \phi$ or $\mathcal{M}, n \models \psi$
- $\mathcal{M}, n \models \phi \mathcal{U}_I \psi$ iff $\exists n' \in n + I$ ($\mathcal{M}, n' \models \psi$ and $\forall n'' \in [n, n') (\mathcal{M}, n'' \models \phi)$)
The future is uncertain!

Sensors are imperfect!

Many sources of uncertainty exists. . .

*Representing and managing uncertainty is important*
Physical Systems and State Estimation

\[ x_t^c \rightarrow \text{Sensor Measurement} \rightarrow y_t \rightarrow y_{tn} \rightarrow \text{State Estimation} \rightarrow x_t \]

\[ x^c(t) \]
\[ x^c(t_n) \]
\[ y_{tn} \]
\[ y_{tn} \text{ 95% prob.} \]
\[ \text{mean}(x_{t|t_n}) \]
\[ \text{mean}(x_{t_n|t_n}) \]
\[ x_{tn} \text{ 95% prob.} \]
UAV inside no-fly-zone?

\[ \text{inside} = \text{false} \]

\[ \Pr(\text{inside}) = 0.1 \]

\[ \Pr(\text{inside}) = 0.4 \]
**Definition (ProbSTL model)**

ProbSTL is defined over model $\mathcal{M} = \langle S, F_B, F_R, F_S, E, S \rangle$

**Definition (ProbSTL Stream)**

A ProbSTL stream $S$ is a tuple of discrete-time signals. The individual signals $x$ are either deterministic $x_n = x_{t_n}$ or stochastic $x_n = \langle x_{t'}|_{t_n}, \ldots \rangle$, $\forall t' \in \mathbb{R}$. Each stochastic variable is defined by its probability distribution $p(x_{t'}|_{t_n}) = p(x_{t'}|y_{t_0}, \ldots, y_{t_n})$.

**Definition (Probabilistic Language $\mathcal{L}_{prob}$)**

\[
\ell ::= \text{const} \mid Pr(E(\tau_p, \ldots, \tau, \ldots)) \mid f_R(\tau, \ldots)
\]

$\tau ::= \tau_d \mid \tau_p$

$\tau_d ::= \ell \mid x_t$

$\tau_p ::= x_{t'|t} \mid f_S(\tau, \ldots)$
UAV altitude under uncertainty

\( \square (\text{altitude}_0 > 3) \)

\( \square (\Pr(\text{altitude}_0 > 3) \geq 0.99) \)

\( \square (\Pr(\text{altitude}_{2|0} > 3) \geq 0.99) \)
Expressivity

*Probabilitic*
- Is the UAV inside the no-fly-zone?

*Introspective*
- Are the predictions reliable?

*Anticipatory*
- Collision in the near future?