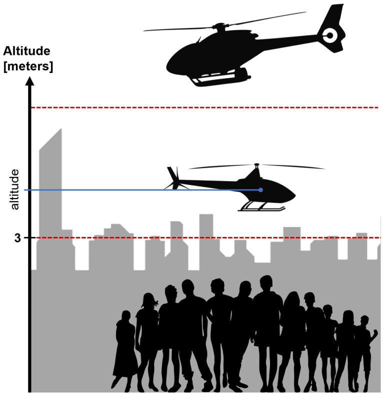
# Probabilistic Signal Temporal Logic for Predictive Stream Reasoning

Mattias Tiger Fredrik Heintz

Artificial Intelligence and Integrated Computer Systems
Department of Computer and Information Science
Linköping University, Sweden

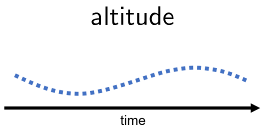
LINKÖPING UNIVERSITY

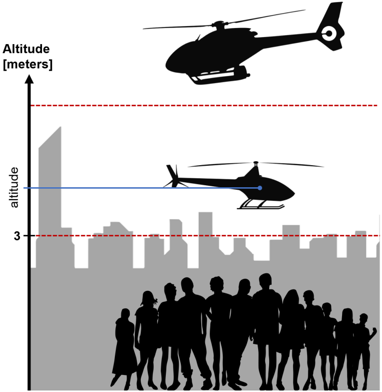
# Unmanned Aerial Vehicle (UAV)



# Stream Reasoning

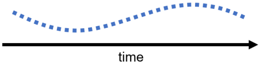
**Stream:**



# Stream Reasoning

altitude



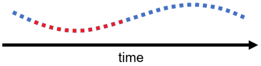

**Global constraint:**

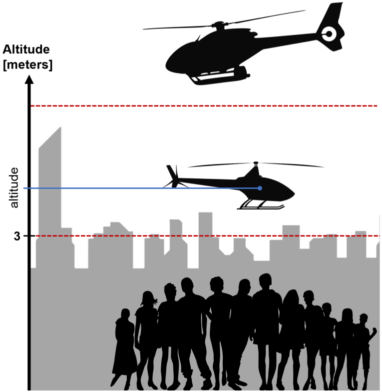
$$\square(\text{altitude} > 3)$$

**True** or **False**

**Monitoring:**

$$\text{altitude} > 3$$



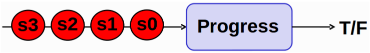



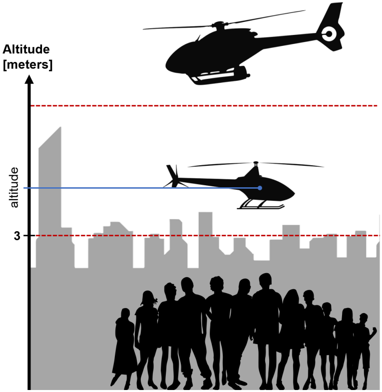
# Stream Reasoning

Metric Temporal Logic (MTL)

$$\Box(\text{altitude} > 3)$$

*Incremental Reasoning using Progression*

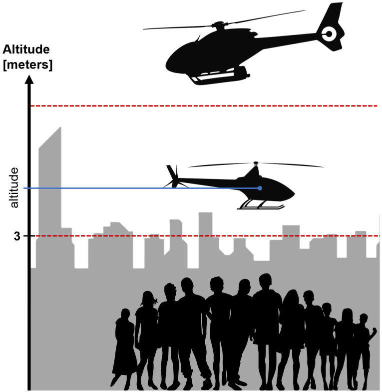


# Reason over predictions of the future

**React when/after it happens:**

$\Box\,(\text{altitude} > 3)$



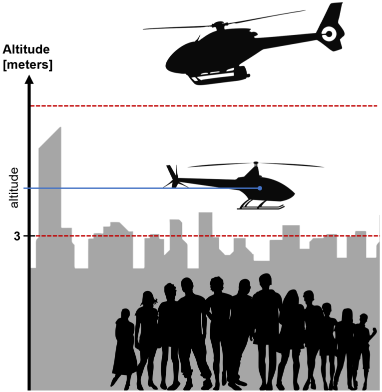
# Reason over predictions of the future

**React when/after it happens:**

$\Box(\text{altitude} > 3)$

**React before it happens:**

$\Box(\text{altitude}_{2|0} > 3)$



# Reason over predictions of the future

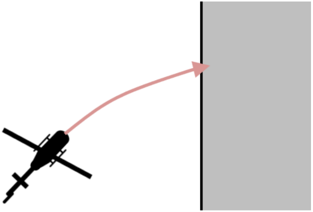
**React when/after it happens:**

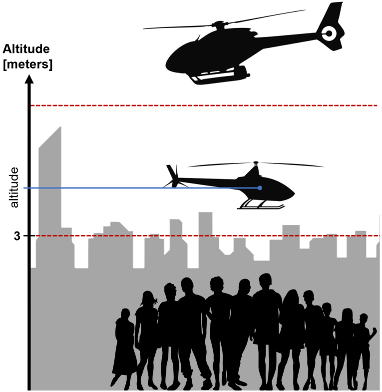
$$\square(\text{altitude} > 3)$$

**React before it happens:**

$$\square(\text{altitude}_{2|0} > 3)$$

**Collision?**





# Predicting the future?

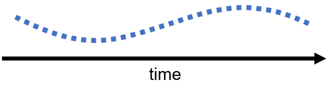
Predicting future states

- What is the source of the stream and its states?
- How are states related over time?

# Streams and Signals

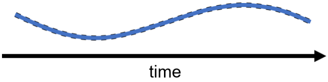
**altitude**

*State Stream*




Discretiation

*Continuous Signal*




Physical Property

$$\text{stream} \models \phi \quad \overset{?}{\Longrightarrow} \quad \text{signal} \models \phi$$

## Definition (STL model)

STL is defined over model $\mathcal{M} = \langle S, F_{\mathbb{B}} \rangle$. $f_p \in F_{\mathbb{B}} : \mathbb{R}^{|S|} \to \{\top, \bot\}$.

## Definition (STL syntax)

$$\phi := \top \mid p \mid \neg\phi \mid \phi \vee \psi \mid \phi\, \mathcal{U}_I\, \psi$$

## Definition (STL semantics)

$$\begin{aligned}
&\mathcal{M}, n \models \top && \\
&\mathcal{M}, n \models p && \text{iff } f_p(S_n) \\
&\mathcal{M}, n \models \neg\phi && \text{iff } \mathcal{M}, n \not\models \phi \\
&\mathcal{M}, n \models \phi \vee \psi && \text{iff } \mathcal{M}, n \models \phi \text{ or } \mathcal{M}, n \models \psi \\
&\mathcal{M}, n \models \phi\, \mathcal{U}_I\, \psi && \text{iff } \exists n' \in n + I \, \big(\mathcal{M}, n' \models \psi \text{ and} \\
& && \quad \forall n'' \in [n, n') \; (\mathcal{M}, n'' \models \phi) \big)
\end{aligned}$$

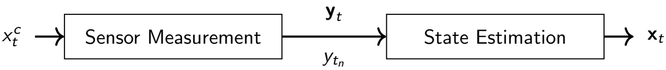
The future is uncertain!

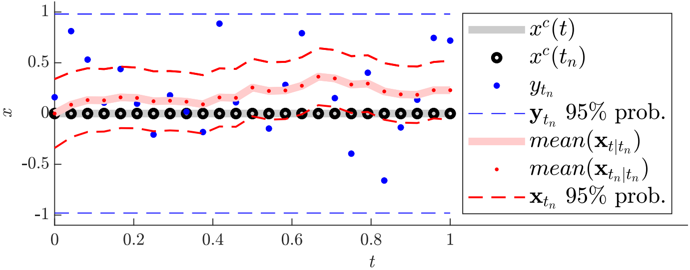
Sensors are imperfect!

Many sources of uncertainty exists. . .

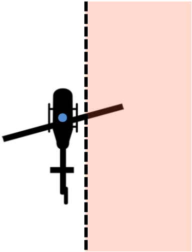
*Representing and managing uncertainty is important*

# Physical Systems and State Estimation

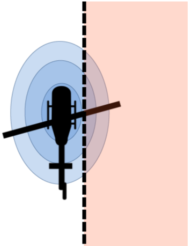


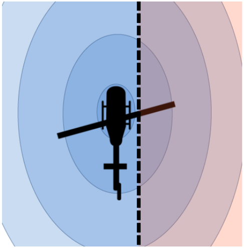
## UAV inside no-fly-zone?



inside $=$ **false**



$Pr(\text{inside}) =$ **0.1**



$Pr(\text{inside}) =$ **0.4**

## Definition (ProbSTL model)

ProbSTL is defined over model $\mathcal{M} = \langle S, F_{\mathbb{B}}, F_{\mathbb{R}}, F_{\mathbb{S}}, \mathbb{E}, \mathbb{S} \rangle$

## Definition (ProbSTL Stream)

A ProbSTL stream $S$ is a tuple of discrete-time signals. The individual signals $x$ are either deterministic $x_n = \mathrm{x}_{t_n}$ or stochastic $x_n = \langle \mathbf{x}_{t'|t_n}, \dots \rangle$, $\forall t' \in \mathbb{R}$. Each stochastic variable is defined by its probability distribution $p(\mathbf{x}_{t'|t_n}) = p(\mathbf{x}_{t'} | y_{t_0}, \dots, y_{t_n})$.

## Definition (Probabilistic Language $\mathcal{L}_{\text{prob}}$)

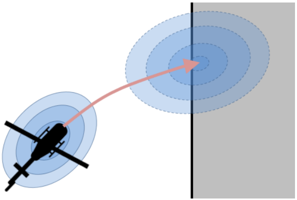
$$\begin{aligned}
\ell &:= \text{const} \mid Pr(E(\tau_p, \dots, \tau, \dots)) \mid f_{\mathbb{R}}(\tau, \dots) \\
\tau &:= \tau_d \mid \tau_p \\
\tau_d &:= \ell \mid \mathrm{x}_t \\
\tau_p &:= \mathrm{x}_{t'|t} \mid f_{\mathbb{S}}(\tau, \dots)
\end{aligned}$$

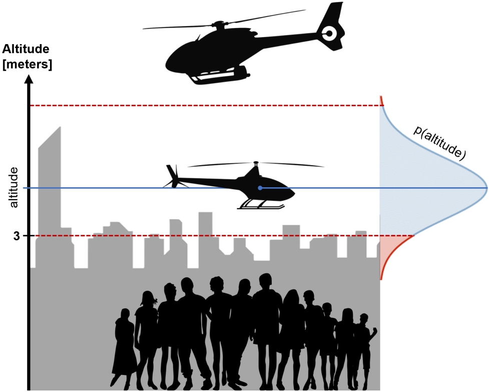
# UAV altitude under uncertainty

$\square\,(\text{altitude}_0 > 3)$

$\square\,(\Pr(\text{altitude}_0 > 3) \geq 0.99)$

$\square\,(\Pr(\text{altitude}_{2|0} > 3) \geq 0.99)$





# Expressivity

*Probabilitic*

- Is the UAV inside the no-fly-zone?

*Introspective*

- Are the predictions reliable?

*Anticipatory*

- Collision in the near future?